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Counter-Compensatory Inter-Vivos Transfers and Parental Altruism: Compatibility or Orthogonality?

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Abstract

The intersection of the *standard* altruism hypothesis with the quite strong evidence that bequests tend to be equal suggests that inter-vivos transfers should be strongly compensatory. Yet the available evidence is not in congruence with this implication. It has therefore been inferred that the motive underlying inter-vivos transfers is not parental altruism. In this paper we present an argument showing why parents who are equally altruistic toward their children optimally transfer *more* to the child whose earnings are higher. We show that rather than being orthogonal to parental altruism, counter-compensating transfers *emanate* from such altruism. A key point in the analysis is that parents and children are interlinked in a rich web of (vertical and possibly horizontal) transfers, reverse transfers, direct transfers, and indirect transfers.

Keywords

Parental altruism, inter-vivos transfers

JEL Classifications

D10, D31, D63, D64

Comments

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Contents

1. Introduction 1

2. The compatibility of equal bequests and parental altruism 3

3. Counter-compensatory inter-vivos transfers and parental altruism 4

4. Conclusions 10

References 12

Mathematical Appendix 13

1. Introduction

Parents are inherently altruistic toward their children. Indeed, parenthood and altruism are intimately intertwined, and it is not possible to bear and rear children nonaltruistically. If, to begin with, we assume that the parents hold and control the entire quantity of the consumption good, altruistic parents must transfer some of the good to their child(ren). The altruism weight attached to the wellbeing of the child determines the size of the transfer. In general, parents can give to their children using inter-vivos transfers and bequests.

When parents have two or more children who are not identical in all relevant respects, the main question of interest is what determines the division of parental giving across children. The auxiliary assumption that altruistic parents are *equally* altruistic toward their children leads to a theoretical prediction that a child who is less "well off" will receive more than a child who is "better off". If wellbeing is measured by earnings, the prediction is that more will be given to the child who earns less (Becker and Tomes, 1979). The available evidence is, however, not easily reconcilable with this prediction.

When we examine the distribution of bequests, it appears that parents typically divide their estate equally among their children. Menchik (1980, 1988) finds that in the division of large estates in Connecticut (1930-1945), equal bequests predominated. Similarly, an examination of wills in Cleveland, Ohio (1964-1965), shows that about 80 percent of siblings shared the estate equally. Wilhelm (1996) finds that in the U.S. (1982), 68.6 percent of parents divided their estates exactly equally among their children, and 88 percent divided their estates approximately equally. Dunn and Phillips (1997) find that in the U.S. (1992), 90 percent of parents named all their children as beneficiaries in their wills, and that about 95 percent of these parents reported that the will "provides about equally" for all the children. In the second section of this paper we explain briefly why equal altruism toward children whose earnings differ is compatible with equal bequests.

The elimination of a degree of freedom with respect to the division of bequests places a heavier burden on inter-vivos transfers. With no recourse to compensating or counter-compensating bequests, inter-vivos transfers become the means of choice to parents (who are equally altruistic toward their children) for affecting the differential earnings of their children. There is some evidence on the distribution of inter-vivos transfers across children. Cox (1987) and Cox and Rank (1992) find that, conditional on a positive transfer having occurred, inter-vivos transfers increased as the income of the recipient increased. The finding is interpreted to be consistent with "an exchange motive" for transfers between parents and children but not with altruism. McGarry (1999) finds that the probability that parents made inter-vivos transfers to their children was related positively and significantly to the children's level of schooling. This finding appears to be at odds with the altruism hypothesis: if schooling is a proxy for permanent income, the probability of a transfer ought to fall with increases in this variable. Altonji, Hayashi, and Kotlikoff (1992) test consumption and income data for the implicit presence of compensatory inter-vivos transfers among extended family members. If transfers are altruistically motivated, within-family consumption differences should be independent of the within-family distribution of income, but the evidence is to the contrary. Similarly, Altonji, Hayashi, and Kotlikoff (1997) find only a modest effect from intergenerational altruism. Specifically, they test a general altruism hypothesis of inter-vivos transfers: a one-dollar increase in the income of parents who are actively engaged in transferring funds to a child coupled with a one-dollar reduction in that child's income would result in a one-dollar increase in the parents' transfer to the child. Their findings reject the altruism hypothesis. Redistributing one dollar from a recipient child to the donor parents leads to less than a 13-cent increase in the parents' inter-vivos transfer to the child. Somewhat surprisingly then, the available evidence does not provide strong support for the prediction that inter-vivos transfers are compensatory. In section 3 we show why this is so: altruistic parents who care equally about their children may well transfer *more* to the child whose earnings are higher. Rather than being orthogonal to parental altruism, counter-compensatory inter-vivos transfers *emanate* from parental altruism.

2. The compatibility of equal bequests and parental altruism

Two characteristics sharply distinguish bequests from inter-vivos transfers: the public nature of bequests, and the finiteness of bequests as the last act of giving to children. For simplicity's sake and without loss of generality we will refer henceforth to the case of two children. We let the earnings of child 1 be larger than the earnings of child 2.[1]

Parents who are equally altruistic toward their children may consider leaving a larger bequest to the lower-earning child 2 (a "compensating" act). However, because the division of bequests is public information, unequal division is tantamount to a public statement that child 2's earnings are relatively low – a declaration that can embarrass child 2. The finite nature of bequests renders them particularly symbolic and significant in the hearts and minds of children as an indicator of the strength of their parents' love. Whereas any particular inter-vivos transfer can be followed by a compensating inter-vivos transfer, a bequest cannot. Child 1, who receives a smaller bequest than child 2, may sense neglect and betrayal and may suffer from envy and

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[1] Behrman (1997) provides evidence that children typically differ in their earnings.

jealousy. While inter-vivos transfers can be made in complete secrecy (not even child 1, let alone outsiders, need know about a transfer that the parents make to child 2) bequests cannot be kept secret; sealed envelopes are distinct from a sealed will. Indeed, Stark (1998) has argued that a rationale for the equal division of bequests is the relative deprivation cost associated with unequal bequests. In general, children constitute a natural and quite cohesive reference group, and tend to engage in intragroup comparisons. These comparisons can give rise to dissatisfaction, dismay, and displeasure. Children refer differently to bequests and to income accruing to them separately from and independent of bequests. The very nature of the accrual of this income implies that when it comes to comparisons within the group of children, such income will be referred to differently than income arising from parental bequests. Relative deprivation will be induced if the parents bequeath to child 1 less than to child 2. As the bequest of child 2 increases relative to the bequest of child 1, child 1's utility declines, as does the utility of the altruistic parents. Altruistic parents trade off this decline in utility against the advantage of providing a larger bequest to child 2. A high cost of relative deprivation prompts the parents to bequeath equally.

3. Counter-compensatory inter-vivos transfers and parental altruism

We study inter-vivos transfers by parents who are equally altruistic toward their two children. Denote by T_i the parental transfer to child i, and by l_i the earnings capability of child i. Thus T_i generates earnings of $l_i T_i$ by child i. We consider first the case in which the earnings capabilities of the children differ: Child 1 whose earnings capability is higher converts transfers to more earnings than does child 2, $l_1 > l_2 \geq 1$. Child 1 attributes his relative success ($l_1/l_2 > 1$) partially to his parents and expresses his gratitude by directing some reverse transfers to them. The sharing coefficient of child 1 with his parents is $r > 0$. The parents may find it optimal to transfer *more* to child 1 than to child 2 since the reverse transfer may amount to more than child 2 would have received. The parents can subsequently give this reverse transfer to child 2 who thus ends up better off.

While a direct parental transfer to child i is transformed by child i into earnings of $l_i T_i$, any subsequent transfer to child *i* is not subject to such a transformation. We may conceive of the parental direct transfer T_i as taking place "at the beginning of the period," of the transformation of T_i into $l_i T_i$ as "production during the period," and of the usage by child 2 of $l_2 T_2$ along with $r\, l_1 T_1$ as his "consumption at the end of the period." Notice that allowing "second round" transfers to child 2 to be subject to a l_2 enhancement will only make our argument stronger.

Suppose that the parents decide to transfer an amount *T* of their income to their children. The parents may transfer *T/2* to the lower-earning child 2. But they can improve on child 2's outcome (which, if he were to receive *T/2*, would be $l_2 T/2$) by solving $l_2 T_2 + r\, l_1 (T - T_2) \geq l_2 T/2$. This gives the critical value[2]

$$T_2^* = \begin{cases} \dfrac{l_2 T/2 - r\, l_1 T}{l_2 - r\, l_1} < T/2 & \text{if } r \leq l_2/2\, l_1 \\ \\ 0 & \text{if } r > l_2/2\, l_1 . \end{cases} \tag{1}$$

From the non-negativity of T_2^* we get that $r < l_2/2 l_1 < 1/2$. With no time discounting and no uncertainty, the parents improve child 2's allocation by directly transferring to him less than half

[2] Detailed derivations of the mathematical results are in an appendix available from the authors upon request.

of the transfer amount. If $r = 0$, we get from the upper part of the equality in (1) that $T_2^* = T/2$, the scenario of an equal division of the transfer amount. Since $\frac{\partial T_2^*}{\partial r} < 0$, the larger is the sharing coefficient of the higher-earning child, the more will the equally altruistic parents deviate from an equal transfer allocation. In addition, from a re-write of the upper part of the equality in (1) we get

$$T_2^* = \frac{T(\frac{1}{2} - r\,\frac{l_1}{l_2})}{1 - r\,\frac{l_1}{l_2}}. \tag{1'}$$

It follows that $\frac{\partial T_2^*}{\partial (l_1/l_2)} < 0$. We thus further infer that the larger the difference between the children's earnings capabilities, the smaller the amount that the equally altruistic parents (initially) transfer to the lower-earning child. Finally, note that even if $l_1 = l_2$ (but $r > 0$), T_2^* is less than $T/2$.

While the magnitude of T_2^* in (1) and the signs of $\partial T_2^* / \partial r$ and $\partial T_2^* / \partial(l_1 / l_2)$ were not derived from an explicit utility maximization, such a maximization happens to give rise to precisely the same magnitude and signs. Let the parents' utility function be

$$U(C_p, C_1, C_2) = (1-a)V_p(\overline{Y}_p - T) + (a/2)V_1(l_1 T_1 - r\,l_1 T_1)$$
$$+ (a/2)V_2(l_2 T_2 + r\,l_1 T_1) \tag{2}$$

where C_i is consumption of $i = p,1,2$; $0 < a/2 < 1/2$ is the weight the parents place on the felicity of each child relative to their own felicity; V_i is the direct pleasure of i from consumption; $\overline{Y}_p$ is the parents' income; and $T = T_1 + T_2$ is the amount the parents transfer out of their income to the children such that T_1 is transferred to child 1, and T_2 is transferred to child 2. We assume henceforth that $V_i(C_i) = ln(C_i)$. The parents maximize their utility with respect to T, T_1, and T_2. This requires differentiating $U(\cdot)$ with respect T_1 and T_2 (or with respect to T and T_1, or with respect to T and T_2). By equating the resulting derivatives to zero and then to each other we obtain optimal values of T_1 and T_2 (and hence of T). In particular, the optimal T_2 is:

$$T_2 = \frac{1}{2}\frac{(l_2 - r\,l_1)\,l_1 T - r\,l_1^2 T}{l_1(l_2 - r\,l_1)} = \frac{T}{2}\,\frac{l_2 - 2r\,l_1}{l_2 - r\,l_1},$$

which is exactly the expression in (1).

Typically, children differ not only in their capabilities to convert transfers into earnings but also in their pre-transfer earnings. How will parents who are equally altruistic toward their children divide the transfer amount between their children when both the difference in earnings and the difference in earnings-capability are considered? Will the parents still transfer to child 2 less than $T/2$?

It is reasonable to assume that the difference in children's earnings (incomes) correlates positively with the difference in their earnings capabilities. We therefore study the case in which the earnings of child 1, E_1, are larger than the earnings of child 2, E_2. The parents' utility function is:

$$U(C_p, C_1, C_2) = (1 - a)V_p(\overline{Y}_p - T) + (a/2)V_1(E_1 + l_1 T_1 - r\,l_1 T_1)$$

$$+ (a/2)V_2(E_2 + l_2 T_2 + r\,l_1 T_1). \quad (3)$$

From the first order condition of the utility maximization problem we get

$$\frac{(1-r)\,l_1}{E_1+(1-r)\,l_1(T-T_2)}=\frac{l_2-r\,l_1}{E_2+l_2T_2+r\,l_1(T-T_2)}$$

or

$$T_2=\frac{1}{2}\frac{(l_2-r\,l_1)[E_1+(1-r)\,l_1T]-(1-r)\,l_1E_2-(1-r)r\,l_1^2T}{(1-r)\,l_1(l_2-r\,l_1)}\equiv\tilde{T}_2\,. \qquad (4)$$

It is instructive to check once again to see whether the optimal transfer to child 2 is less than T/2. Comparing $\tilde{T}_2$ to $T/2$ gives that $\tilde{T}_2 < T/2$ if $\frac{l_2-r\,l_1}{l_1-r\,l_1}<\frac{E_2}{E_1}$. Assuming that $E_1/E_2 = l_1/l_2$ gives that $\tilde{T}_2 < T/2$ if $l_1^2 > l_1/l_2$, which indeed holds. Inter-vivos transfers are still counter-compensatory.

Suppose that $E_1 > E_2$, but that $l_1 = l_2 = l$; there is a difference in earnings not accompanied by a difference in earnings capabilities. From the equality in (4) the optimal T_2 is now

$$T_2=\frac{1}{2}\frac{E_1-E_2+(1-2r)\,l\,T}{l(1-r)}\equiv\hat{T}_2\,. \qquad (5)$$

Is $\hat{T}_2 < T/2$? It turns out that $\hat{T}_2<\frac{T}{2}$ iff $\frac{E_1-E_2}{r\,l}<T$. Since this inequality holds for all sufficiently large T's, we conclude once again that counter-compensating inter-vivos transfers can arise.

Finally, we note that when $r = 0$ and $l_i=1$, $i=1,2$, T_2 in (4) reduces to

$$T_2 = \begin{cases} \dfrac{E_1 - E_2}{2} + \dfrac{T}{2} & if \quad T > E_1 - E_2 \\ \\ T & if \quad T \leq E_1 - E_2 \; . \end{cases} \tag{6}$$

Here $T_2 > T/2$ iff $E_1 > E_2$ and $T_2 = T/2$ iff $E_1 = E_2$. Only in this special case are inter-vivos transfers (by parents who are equally altruistic toward their children) unequivocally compensatory. (This is the case alluded to by Becker and Tomes.)

Note that the sequence studied hitherto is not the only sequence that supports non-equal division of the transfer amount by parents who care for their children equally. If the higher-earning child is altruistic toward the lower-earning child, the parents can again improve on child 2's outcome by solving $l_2T_2 + \tilde{r}\, l_1(T - T_2) = l_2T/2$ where $\tilde{r}$ is the altruism coefficient of child 1 toward child 2. The solution of this problem is identical to (1), and it likewise follows that the larger the altruism coefficient of child 1 toward child 2, and the larger the difference in the earnings capabilities of the two children, the larger the parents' deviation from the equal division of the transfer.

In both schemes, parents who are equally altruistic toward their two children choose to transfer *less* to the child whose pre-transfer earnings and/or earnings capability are lower. Both schemes hinge on the gratitude or the altruism of child 1 and on transfers from that child. The two schemes differ though. In the first scheme the parents directly transfer less than half of the transfer amount to child 2. Yet, directly (as originators), *and* indirectly (as intermediaries), they end up transferring no less to child 2 than to child 1. In the second scheme, the *parental* transfer to child 2 is less than half the transfer amount.

The two transfer schemes point to an additional relationship: the larger the difference in the earnings capabilities of the two children, the smaller the sharing coefficient (r) or the altruism coefficient ($\tilde{r}$) required to sustain the schemes. Re-arranging the upper part of the equality in (1) gives

$$r = \frac{1}{l_1 / l_2} \frac{T - 2T_2^*}{2(T - T_2^*)} < 1 \ . \qquad (1'')$$

From (1") we get that $\frac{\partial r}{\partial(l_1 / l_2)} < 0$; even a small sharing coefficient suffices to support counter-compensating inter-vivos transfers when the difference between the children's earnings is large. If the likelihood of the existence of r is inversely related to its magnitude, the likelihood of counter-compensatory inter-vivos transfers is *larger*, not smaller, when the difference between the children's earnings capabilities is larger. An identical relationship holds for $\tilde{r}$, that is, $\frac{\partial \tilde{r}}{\partial(l_1 / l_2)} < 0$.

4. Conclusions

The intersection of the *standard* altruism hypothesis with the quite strong evidence that bequests tend to be equal suggests that inter-vivos transfers should be strongly compensatory. Yet the available evidence is not in congruence with this implication. It has therefore been inferred that the motive underlying inter-vivos transfers is not parental altruism. This conclusion appears to be unwarranted: given equal bequests, there is no inherent contradiction between parental altruism and noncompensating inter-vivos transfers. In fact, parental altruism can well mandate inter-vivos transfers that are anything but compensatory. We argue that parents who are equally altruistic toward their children will optimally transfer *more* to the child whose pre-transfer earnings and/or earnings capability are higher. We refer to such transfer behavior as counter-compensatory, and demonstrate that rather than being orthogonal to parental altruism, noncompensating transfers *emanate* from such altruism. Moreover, an observation that larger parental transfers are made to the child whose earnings are higher cannot be construed as evidence in support of "exchange" as opposed to altruism. Such an observation may well underscore the role of altruism; altruistic parents rely on inter-sibling altruism. Our key point is that there is more to inter-vivos transfers than what the eye meets first. Parents and children are interlinked in a rich web of (vertical and possibly horizontal) transfers, reverse transfers, direct transfers, and indirect transfers. This multitude of transfers occurs over the entire lifetime of the parents and their children. Therefore, a proper test of parental altruism requires observations on lifetime longitudinal transfers between parents and their children as well as on transfers between the children. Currently available data sets do not appear to facilitate such encompassing observations.

References

Altonji, Joseph G., Hayashi, Fumio and Kotlikoff, Laurence J. “Is the Extended Family Altruistically Linked? Direct Tests Using Microdata.” *American Economic Review*, December 1992, 82(5), pp. 1177 - 1198.

_______. “Parental Altruism and Inter Vivos Transfers: Theory and Evidence.” *Journal of Political Economy*, December 1997, 105(6), pp. 1121 - 1166.

Becker, Gary S. and Tomes, Nigel. “An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility.” *Journal of Political Economy,* December 1979, 87(6), pp. 1153 - 1189.

Behrman, Jere R. “Intrahousehold Distribution and the Family.” In Mark R. Rosenzweig and Oded Stark (eds.) *Handbook of Population and Family Economics*. Amsterdam: North-Holland, 1997.

Cox, Donald. “Motives for Private Transfers.” *Journal of Political Economy,* June 1987, 95(3), pp. 508 - 546.

Cox, Donald and Rank, Mark R. “Inter-vivos Transfers and Intergenerational Exchange.” *Review of Economics and Statistics*, May 1992, 74(2), pp. 305 - 314.

Dunn, Thomas A. and Phillips, John W. “The Timing and Division of Parental Transfers to Children.” *Economics Letters*, 1997, 54, pp. 135 - 137.

McGarry, Kathleen. “Inter Vivos Transfers and Intended Bequests.” *Journal of Public Economics* 73, pp. 321-351.

Menchik, Paul L. “Primogeniture, Equal Sharing, and the U.S. Distribution of Wealth.” *Quarterly Journal of Economics,* March 1980, 94, pp. 299 - 316.

_______. “Unequal Estate Division: Is It Altruism, Reverse Bequests, or Simply Noise?” in Denis Kessler and Andrew Masson (eds.), *Modelling the Accumulation and Distribution of Wealth.* Oxford: Oxford University Press, 1988.

Stark, Oded. “Equal Bequests and Parental Altruism: Compatibility or Orthogonality?” *Economics Letters*, 1998, 60, pp. 167-171.

Wilhelm, Mark O. “Bequest Behavior and the Effect of Heirs’ Earnings: Testing the Altruistic Model of Bequests.” *American Economic Review*, September 1996, 86(4), pp. 874 - 892.

Counter-Compensatory Inter-Vivos Transfers and Parental Altruism: Compatibility or Orthogonality?

Mathematical Appendix

The parents solve

$$\lambda_2 T_2 + \rho\lambda_1(T - T_2) \geq \lambda_2 T/2.$$

It follows that

$$\begin{aligned}
2\lambda_2 T_2 - 2\rho\lambda_1 T_2 &\geq \lambda_2 T - 2\rho\lambda_1 T \\
(\lambda_2 - \rho\lambda_1)2T_2 &\geq \lambda_2 T - 2\rho\lambda_1 T \\
T_2 &\geq \frac{\lambda_2 T/2 - \rho\lambda_1 T}{\lambda_2 - \rho\lambda_1} \text{ if } \lambda_2 - \rho\lambda_1 > 0 \\
T_2 &\leq \frac{\lambda_2 T/2 - \rho\lambda_1 T}{\lambda_2 - \rho\lambda_1} \text{ if } \lambda_2 - \rho\lambda_1 < 0.
\end{aligned}$$

Therefore,

$$T_2^* = \frac{\lambda_2 T/2 - \rho\lambda_1 T}{\lambda_2 - \rho\lambda_1} \tag{1}$$

is the critical value.

Now derive the partial derivative $\partial T_2/\partial\rho$:

$$\begin{aligned}
T_2 &= \frac{\lambda_2 T/2 - \rho\lambda_1 T}{\lambda_2 - \rho\lambda_1} \\
&= T\frac{\lambda_2/2 - \rho\lambda_1}{\lambda_2 - \rho\lambda_1} \\
\frac{\partial T_2}{\partial\rho} &= T\frac{(\lambda_2/2 - \rho\lambda_1)\lambda_1 - (\lambda_2 - \rho\lambda_1)\lambda_1}{(\lambda_2 - \rho\lambda_1)^2} \\
&= \frac{-T\lambda_1\lambda_2/2}{(\lambda_2 - \rho\lambda_1)^2} < 0.
\end{aligned}$$

Also derive the partial derivative $\partial T_2/\partial(\lambda_1/\lambda_2)$:

$$T_2 = T\frac{\frac{1}{2} - \rho\frac{\lambda_1}{\lambda_2}}{1 - \rho\frac{\lambda_1}{\lambda_2}} \tag{1'}$$

$$\begin{aligned}
\frac{\partial T_2}{\partial\frac{\lambda_1}{\lambda_2}} &= T\frac{\left(\frac{1}{2} - \rho\frac{\lambda_1}{\lambda_2}\right)\rho - \left(1 - \rho\frac{\lambda_1}{\lambda_2}\right)\rho}{\left(1 - \rho\frac{\lambda_1}{\lambda_2}\right)^2} \\
&= \frac{-\frac{1}{2}T\rho}{\left(1 - \rho\frac{\lambda_1}{\lambda_2}\right)^2} < 0.
\end{aligned}$$

Now, consider the utility maximization problem of the parents:

$$\underset{T,T_1,T_2}{\text{Max}}\; U(C_p, C_1, C_2) = (1-\alpha)V_p\left(\overline{Y}_p - T\right) + (\alpha/2)V_1(\lambda_1 T_1 - \rho\lambda_1 T_1)$$
$$+ (\alpha/2)V_2(\lambda_2 T_2 + \rho\lambda_1 T_1) \tag{2}$$

subject to $T = T_1 + T_2$. It can be equivalently written as

$$\underset{T,T_2}{\text{Max}}\; U(C_p, C_1, C_2) = (1-\alpha)V_p\left(\overline{Y}_p - T\right) + (\alpha/2)V_1((\lambda_1 - \rho\lambda_1)(T - T_2))$$
$$+ (\alpha/2)V_2(\rho\lambda_1 T + (\lambda_2 - \rho\lambda_1)T_2). \tag{A1}$$

The FOCs of this problem are

$$\left(\frac{\alpha}{2}\right)[V_2'(\rho\lambda_1 T + (\lambda_2 - \rho\lambda_1)T_2)(\lambda_2 - \rho\lambda_1) - V_1'((\lambda_1 - \rho\lambda_1)(T - T_2))(\lambda_1 - \rho\lambda_1)] = 0$$

and

$$(\alpha/2)\rho\lambda_1 V_2'(\rho\lambda_1 T + (\lambda_2 - \rho\lambda_1)T_2) - (1-\alpha)V_p'\left(\overline{Y}_p - T\right)$$
$$-(\alpha/2)(\lambda_1 - \rho\lambda_1)V_1'((\lambda_1 - \rho\lambda_1)(T - T_2)) = 0$$

Considering the case $V(C_i) = \ln(C_i)$, the FOCs can be rewritten as

$$\frac{\lambda_2 - \rho\lambda_1}{\rho\lambda_1 T + (\lambda_2 - \rho\lambda_1)T_2} - \frac{1}{(T - T_2)} = 0 \tag{A2}$$

$$\frac{(\alpha/2)\rho\lambda_1}{\rho\lambda_1 T + (\lambda_2 - \rho\lambda_1)T_2} - \frac{1-\alpha}{\overline{Y}_p - T} - \frac{(\alpha/2)}{T - T_2} = 0. \tag{A3}$$

Solving (A2), we have

$$(T - T_2)(\lambda_2 - \rho\lambda_1) = \rho\lambda_1 T + (\lambda_2 - \rho\lambda_1)T_2$$
$$T[\lambda_2 - 2\rho\lambda_1] = T_2 2(\lambda_2 - \rho\lambda_1)$$
$$T_2 = \frac{T}{2}\frac{\lambda_2 - 2\rho\lambda_1}{\lambda_2 - \rho\lambda_1}.$$

If the parents consider the difference in the children's earnings as well as the difference in the children's earnings capability, the parents' maximization problem is modified as

$$\underset{T,T_1,T_2}{\text{Max}}\; U(C_p, C_1, C_2) = (1-\alpha)V_p\left(\overline{Y}_p - T\right) + (\alpha/2)V_1(E_1 + \lambda_1 T_1 - \rho\lambda_1 T_1)$$
$$+ (\alpha/2)V_2(E_2 + \lambda_2 T_2 + \rho\lambda_1 T_1) \tag{3}$$

subject to $T = T_1 + T_2$. Again, it can be equivalently written as

$$\underset{T,T_2}{\text{Max}}\; U(C_p, C_1, C_2) = (1-\alpha)V_p\left(\overline{Y}_p - T\right) + (\alpha/2)V_1(E_1 + (\lambda_1 - \rho\lambda_1)(T - T_2))$$
$$+ (\alpha/2)V_2(E_2 + \rho\lambda_1 T + (\lambda_2 - \rho\lambda_1)T_2). \tag{A4}$$

The corresponding FOCs are

$$\frac{\lambda_2 - \rho\lambda_1}{E_2 + \rho\lambda_1 T + (\lambda_2 - \rho\lambda_1)T_2} - \frac{(1-\rho)\lambda_1}{E_1 + (1-\rho)\lambda_1(T - T_2)} = 0 \qquad \text{(A5)}$$

and

$$\frac{(\alpha/2)\rho\lambda_1}{E_2 + \rho\lambda_1 T + (\lambda_2 - \rho\lambda_1)T_2} - \frac{1-\alpha}{\overline{Y}_p - T} - \frac{(\alpha/2)(\lambda_1 - \rho\lambda_1)}{E_1 + (\lambda_1 - \rho\lambda_1)(T - T_2)} = 0. \qquad \text{(A6)}$$

Now we try to solve T_2 from (A5):

$$\frac{\lambda_2 - \rho\lambda_1}{E_2 + \rho\lambda_1 T + (\lambda_2 - \rho\lambda_1)T_2} = \frac{(1-\rho)\lambda_1}{E_1 + (1-\rho)\lambda_1(T - T_2)}$$

$$[E_1 + (1-\rho)\lambda_1(T - T_2)](\lambda_2 - \rho\lambda_1) = (1-\rho)\lambda_1[E_2 + \rho\lambda_1 T + (\lambda_2 - \rho\lambda_1)T_2]$$

$$[E_1 + (1-\rho)\lambda_1 T - (1-\rho)\lambda_1 T_2](\lambda_2 - \rho\lambda_1) = (1-\rho)\lambda_1[E_2 + \rho\lambda_1 T + (\lambda_2 - \rho\lambda_1)T_2]$$

$$[E_1 + (1-\rho)\lambda_1 T](\lambda_2 - \rho\lambda_1) - (1-\rho)\lambda_1[E_2 + \rho\lambda_1 T] = (1-\rho)\lambda_1 T_2(\lambda_2 - \rho\lambda_1) + (1-\rho)\lambda_1(\lambda_2 - \rho\lambda_1)T_2$$

$$[E_1 + (1-\rho)\lambda_1 T](\lambda_2 - \rho\lambda_1) - (1-\rho)\lambda_1[E_2 + \rho\lambda_1 T] = T_2\{(1-\rho)\lambda_1(\lambda_2 - \rho\lambda_1) + (1-\rho)\lambda_1(\lambda_2 - \rho\lambda_1)\}$$

$$\begin{aligned} T_2 &= \frac{[E_1 + (1-\rho)\lambda_1 T](\lambda_2 - \rho\lambda_1) - (1-\rho)\lambda_1[E_2 + \rho\lambda_1 T]}{2(1-\rho)\lambda_1(\lambda_2 - \rho\lambda_1)} \\ &= \frac{[E_1 + (1-\rho)\lambda_1 T](\lambda_2 - \rho\lambda_1) - (1-\rho)\lambda_1 E_2 - (1-\rho)\lambda_1^2\rho T}{2(1-\rho)\lambda_1(\lambda_2 - \rho\lambda_1)} \equiv \tilde{T}_2. \end{aligned} \qquad (4)$$

We have $\tilde{T}_2 < T/2$ if

$$\frac{[E_1 + (1-\rho)\lambda_1 T](\lambda_2 - \rho\lambda_1) - (1-\rho)\lambda_1 E_2 - (1-\rho)\lambda_1^2\rho T}{2(1-\rho)\lambda_1(\lambda_2 - \rho\lambda_1)} < \frac{T}{2}$$

$$\frac{[E_1 + (1-\rho)\lambda_1 T](\lambda_2 - \rho\lambda_1) - (1-\rho)\lambda_1 E_2 - (1-\rho)\lambda_1^2\rho T}{(1-\rho)\lambda_1(\lambda_2 - \rho\lambda_1)} < T$$

$$\begin{aligned} [E_1 + (1-\rho)\lambda_1 T](\lambda_2 - \rho\lambda_1) - (1-\rho)\lambda_1 E_2 - (1-\rho)\lambda_1^2\rho T &< T(1-\rho)\lambda_1(\lambda_2 - \rho\lambda_1) \\ &= T(1-\rho)(\lambda_1\lambda_2 - \rho\lambda_1^2) \\ &= T(1-\rho)\lambda_1\lambda_2 - T(1-\rho)\rho\lambda_1^2 \\ [E_1 + (1-\rho)\lambda_1 T](\lambda_2 - \rho\lambda_1) - (1-\rho)\lambda_1 E_2 &< T(1-\rho)\lambda_1\lambda_2 \\ (\lambda_2 - \rho\lambda_1)E_1 + (1-\rho)\lambda_1\lambda_2 T - (1-\rho)\rho\lambda_1^2 T - (1-\rho)\lambda_1 E_2 &< (1-\rho)\lambda_1\lambda_2 T \\ (\lambda_2 - \rho\lambda_1)E_1 &< (1-\rho)\lambda_1 E_2 + (1-\rho)\rho\lambda_1^2 T \\ &= (1-\rho)\lambda_1[E_2 + \rho\lambda_1 T]. \end{aligned}$$

Since $(1-\rho)\lambda_1 E_2 < (1-\rho)\lambda_1[E_2 + \rho\lambda_1 T]$, $\tilde{T}_2 < T/2$ is implied by

$$\begin{aligned} (\lambda_2 - \rho\lambda_1)E_1 &< (1-\rho)\lambda_1 E_2 \\ \frac{E_1}{E_2} &< \frac{(1-\rho)\lambda_1}{\lambda_2 - \rho\lambda_1}. \end{aligned}$$

Assuming $E_1/E_2 = \lambda_1/\lambda_2$, this sufficient condition for $\tilde{T}_2 < T/2$ becomes

$$\frac{\lambda_1}{\lambda_2} < \frac{(1-\rho)\lambda_1}{\lambda_2 - \rho\lambda_1}$$

$$\lambda_1\lambda_2 - \rho\lambda_1^2 < \lambda_1\lambda_2 - \rho\lambda_1\lambda_2, \text{ if } \lambda_2 - \rho\lambda_1 > 0 \text{ and } \lambda_2 > 0$$
$$\lambda_1^2 > \lambda_1\lambda_2.$$

Suppose $E_1 > E_2$ and $\lambda_1 = \lambda_2 = \lambda$. Then

$$T_2 = \frac{[E_1 + (1-\rho)\lambda_1 T](\lambda_2 - \rho\lambda_1) - (1-\rho)\lambda_1 E_2 - (1-\rho)\lambda_1^2\rho T}{2(1-\rho)\lambda_1(\lambda_2 - \rho\lambda_1)}$$

reduces to

$$\begin{aligned} T_2 &= \frac{[E_1 + (1-\rho)\lambda T](1-\rho)\lambda - (1-\rho)\lambda E_2 - (1-\rho)\lambda^2\rho T}{2(1-\rho)^2\lambda^2} \\ &= \frac{[E_1 + (1-\rho)\lambda T] - E_2 - \lambda\rho T}{2(1-\rho)\lambda} \\ &= \frac{E_1 - E_2 + (1-2\rho)\lambda T}{2(1-\rho)\lambda} \equiv \hat{T}_2. \end{aligned} \tag{5}$$

Therefore, $\hat{T}_2 < T/2$ iff

$$\begin{aligned} \frac{E_1 - E_2 + (1-2\rho)\lambda T}{2(1-\rho)\lambda} &< \frac{T}{2} \\ E_1 - E_2 + (1-2\rho)\lambda T &< T(1-\rho)\lambda, \text{ if } (1-\rho)\lambda > 0 \\ E_1 - E_2 &< T\rho\lambda \\ \frac{E_1 - E_2}{\rho\lambda} &< T. \end{aligned}$$

Suppose $\rho = 0$ and $\lambda_i = 1, i = 1,2$. Then (4) reduces to

$$\begin{aligned} T_2 &= \frac{[E_1 + T] - E_2}{2} \\ &= \frac{E_1 - E_2}{2} + \frac{T}{2}. \end{aligned} \tag{6}$$

If the higher-earnings child is altruistic toward the lower-earnings child, the parents can improve on child 2's outcome by solving

$$\lambda_2 T_2 + \tilde{\rho}\lambda_1(T - T_2) = \lambda_2 T/2. \tag{A7}$$

The solution is

$$\begin{aligned} T_2 &= \frac{1}{2}T\frac{\lambda_2 - 2\tilde{\rho}\lambda_1}{\lambda_2 - \tilde{\rho}\lambda_1} \\ &= \frac{\lambda_2 T/2 - \tilde{\rho}\lambda_1 T}{\lambda_2 - \tilde{\rho}\lambda_1} \end{aligned}$$

which is identical to (1).

Recall that the upper part of (1) is

$$T_2^* = \frac{\lambda_2 T/2 - \rho\lambda_1 T}{\lambda_2 - \rho\lambda_1}$$

$$
\begin{aligned}
T_2^*(\lambda_2 - \rho\lambda_1) &= \lambda_2 T/2 - \rho\lambda_1 T \\
T_2^*\lambda_2 - \lambda_2 T/2 &= T_2^*\rho\lambda_1 - \rho\lambda_1 T \\
\lambda_2(T_2^* - T/2) &= \rho\lambda_1(T_2^* - T) \\
\rho &= \frac{\lambda_2(T_2^* - T/2)}{\lambda_1(T_2^* - T)} \\
&= \frac{1}{\lambda_1/\lambda_2} \cdot \frac{2T_2^* - T}{2(T_2^* - T)} < 1. \qquad (1'')
\end{aligned}
$$

From the expression of ρ in $(1'')$, we get that

$$\frac{\partial\rho}{\partial(\lambda_1/\lambda_2)} = \frac{-1}{(\lambda_1/\lambda_2)^2} \cdot \frac{2T_2^* - T}{2(T_2^* - T)} < 0.$$

From (A7), we can obtain equivalent expressions for $\bar{\rho}$ and $\partial\bar{\rho}/\partial(\lambda_1/\lambda_2)$. Thus, we have $\partial\bar{\rho}/\partial(\lambda_1/\lambda_2) < 0$ as well.

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